Astrophysical constraints for the quantum gravity effects

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Where quantum gravity effects are expected to be significant?

Relevant constants of nature are here $G$ (gravity), $\hbar$ (quantum), $c$ (relativity). Theory of QG should depends on these constants. Performing dimensional analysis one can find scale relevant for QG effects

\[
\begin{align*}
    l_{Pl} &= \sqrt{\frac{\hbar G}{c^3}} = 1.6 \times 10^{-35} \text{ m} \\
    t_{Pl} &= \sqrt{\frac{\hbar G}{c^5}} = 5.4 \times 10^{-44} \text{ s} \\
    E_{Pl} &= \sqrt{\frac{\hbar c^5}{G}} = 1.2 \times 10^{19} \text{ GeV}
\end{align*}
\]

Do we know the physics beyond the Planck scale? We have candidates like loop quantum gravity, LQG for short.
Atoms of space

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Astrophysical constraints for the quantum gravity effects
A little more than 100 years ago most people - and most scientists - thought of matter as continuous. Although since ancient times some philosophers and scientists had speculated that if matter were broken up into small enough bits, it might turn out to be made up of very tiny atoms, few thought the existence of atoms could ever be proved. Then Einstein (1905) and Smoluchowski (1906) independently presented a way to indirectly confirm the existence of atoms and molecules by study of Brownian motions.

Theory of Einstein and Smoluchowski has been proved experimentally by Perrin in 1908.
A main obstacle in formulating quantum theory of gravitational interactions is the lack of any empirical clue. For now there is no empirical evidence of quantum gravity. Since quantum gravity effects are expected to be significant at the energies approaching $10^{19}$ GeV, direct experimental probing becomes inaccessible. Therefore alternative methods of investigation ought to be taken into account. The most promising involve observations of the cosmos.

LHC reach 14 TeV (presently up to 1 TeV at Tevatron) while we need $10^{16}$ TeV (Planck energy). We never reach it with the present generation of accelerators.

It is like probing atomic structure with Earth size resolution devices!

Other possibilities? → "Indirect methods"
Indirect methods

Because we are not able to create sufficient conditions in the laboratories on Earth, we have to look for them in space.

Cosmology

- LQG $\rightarrow$ Big Bounce $\rightarrow$ primordial perturbations $\rightarrow$ CMB
- Relic gravitational waves $\rightarrow$ B-type polarization of CMB

Violation of Lorentz Invariance

- Modified dispersion relation $\rightarrow$ short-duration GRB
- Threshold anomalies $\rightarrow$ effects on GKZ cutoff

Gravitational collapse

- Threshold mass for black holes $\rightarrow$ cutoff on the small primordial black holes?
Background dynamics → Effective Friedmann equation:

\[ H^2 \equiv \left( \frac{1}{2\bar{p}} \frac{d\bar{p}}{dt} \right)^2 = \frac{8\pi G}{3} \rho \left( 1 - \frac{\rho}{\rho_c} \right) \]
We perturb basic variables around a spatially flat FRW background

\[ E = \bar{E} + \delta E \]
\[ A = \bar{A} + \delta A \]

Perturbations \((\delta A, \delta E)\) can be splitted for the

- scalar (coupled with a scalar field - fairly complicated system)
- vector (simple but less interesting - decaying modes)
- tensor (gravitational waves - relatively simple)

\[ \frac{d^2}{d\eta^2} h^i_a + 2\bar{k} \frac{d}{d\eta} h^i_a - \nabla^2 h^i_a + \tilde{T}_Q h^i_a = 0 \]
Tensor power spectrum

Tensor power spectra from the Big Bounce $^1$:

\[ \mathcal{P}_T \]

IR \hspace{2cm} UV

\( t=50[l_{Pl}] \)

Correlation function:

\[ \langle 0 | \hat{h}^a_b(x, \eta) \hat{h}^b_a(y, \eta) | 0 \rangle = \int \frac{dk}{k} \mathcal{P}_T(k, \eta) \frac{\sin kr}{kr} \]

More realistic evolution: Big Bounce + Inflation scenario

Since now we have considered only isolated bounce phase. However further stage of evolution, namely inflation, should be also taken into account.

In order to produce inflation we consider a model with massive potential

\[ V(\phi) = \frac{m^2}{2} \phi^2 \]

then EOM is given by

\[ \ddot{\phi} + 3H\dot{\phi} + m^2 \phi = 0. \]
### How generic is inflation in the B+I scenario?

**Table:** Values of $\bar{\phi}_{\text{max}}$ for the different $P(t_0)$ and $m$ (all parameters in Planck units).

<table>
<thead>
<tr>
<th>$P(t_0)/m$</th>
<th>1</th>
<th>$10^{-1}$</th>
<th>$10^{-2}$</th>
<th>$10^{-3}$</th>
<th>$10^{-4}$</th>
<th>$10^{-5}$</th>
<th>$10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.8</td>
<td>1.1</td>
<td>1.4</td>
<td>1.8</td>
<td>2.1</td>
<td>2.2</td>
</tr>
<tr>
<td>$10^{-1}$</td>
<td>0.9</td>
<td>1.1</td>
<td>1.5</td>
<td>1.8</td>
<td>2.2</td>
<td>2.4</td>
<td>2.7</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>0.7</td>
<td><strong>1.6</strong></td>
<td>1.8</td>
<td>2.2</td>
<td>2.5</td>
<td>2.8</td>
<td>3.0</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>1.3</td>
<td>2.2</td>
<td>2.5</td>
<td>2.9</td>
<td>3.2</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>$10^{-4}$</td>
<td>2.0</td>
<td>3.0</td>
<td>3.2</td>
<td>3.6</td>
<td>4.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{-5}$</td>
<td>2.7</td>
<td><strong>3.7</strong></td>
<td>3.9</td>
<td>4.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$10^{-6}$</td>
<td>3.4</td>
<td><strong>4.4</strong></td>
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<td>$10^{-7}$</td>
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</tr>
</tbody>
</table>

**Conclusions:** Phase of Big Bounce can easily set the proper initial conditions for inflation! The parameters of inflation are weakly sensitive on the initial conditions from the contracting phase!
Simple constraints from the inflation

The condition $\rho \leq \rho_c$ implies $|\bar{\varphi}_{\text{max}}| \leq \frac{\sqrt{2}\rho_c}{m}$. The parameter $\bar{\varphi}_{\text{max}}$ is related to e-folding number for inflation, $N \simeq 2\pi \frac{\bar{\varphi}^2_{\text{max}}}{m^2_{\text{Pl}}}$. Taking the standard inflationary parameters (which can be partially fixed from the CMB observations) we find observational constraint on the Barbero-Immirzi parameter

$$\gamma \leq 1222.$$ 

We have used here $\rho_c = \frac{\sqrt{3}}{16\pi^2\gamma^3} m^4_{\text{Pl}}$ (the $\bar{\mu}$-scheme in LQC). Alternatively one can take $\rho_c = \frac{3}{8\pi\gamma^2\lambda^2} m^2_{\text{Pl}}$ where $\lambda$ is scale of quantum polymerization. Based on this we find

$$\lambda \leq 7 \cdot 10^4 l_{\text{Pl}}.$$ 

Here we have used value of Barbero-Immirzi parameter calculated by Meissner, $\gamma = \gamma_M = 0.2375$. 
Towards testing the quantum cosmology models

Work in collaboration with Michał Kamionka, Aleksandra Kurek and Marek Szydłowski.

The standard inflationary spectrum is parametrised in the power-law form

$$P_1(k) = A_s \left( \frac{k}{k_0} \right)^{n_s-1}.$$  

The modifications due to the Big Bounce can be introduced by the additional prefactor $\Delta(k, k_*)$. Then power spectrum takes the form

$$P_2(k) = \Delta(k, k_*) A_s \left( \frac{k}{k_0} \right)^{n_s-1}.$$
The bounce-factor $\Delta(k, k_*)$ can have the form

$$\Delta(k, k_*) = 1 + \frac{k_*^4}{2k^4} \left[ 1 + \cos \left( \frac{2k}{k_*} \right) \left( -1 + \frac{2k^2}{k_*^2} \right) - \frac{2k}{k_*} \sin \left( \frac{2k}{k_*} \right) \right]$$

This shape is generic for the Big Bounce models. In the simplified form we can consider

$$\Delta(k, k_*) = 1 - \frac{\sin \left( \frac{3k}{2k_*} \right)}{\left( \frac{3k}{2k_*} \right)}.$$
Parameter $k_*$ is related with the value of the Hubble factor at the beginning of inflation.
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Astrophysical constraints for the quantum gravity effects
Quantum gravity effects manifest themselves as a violation of the Lorentz invariance. The modified relation of dispersion for the photons can be written as follows

\[ p^2 = E^2 \left( 1 + \frac{E}{M_{QG}} \right). \]

This implies energy-dependent speed of light

\[ v_{\text{gr}} = \frac{\partial E}{\partial p} \simeq 1 - \frac{E}{M_{QG}}. \]

Therefore we expect delays in arrivals of photons

\[ \Delta t_{\text{LV}} = \frac{\Delta E}{M_{QG}} L = \frac{\Delta E}{M_{QG}} H_0 \int_0^z \frac{dz'}{H(z')} . \]
Astrophysical constraints for the quantum gravity effects
Beside the effect of LV we expect also unknown intrinsic energy-dependent time-lags. We take this possibility into account by inclusion of a term \( b_{bf} \) specified in the rest frame of the source. Then observed time-lag is given by

\[
\Delta t_{\text{obs}} = \Delta t_{\text{LV}} + b_{sf}(1 + z)
\]

what can be rewritten to the linear fitting function

\[
\frac{\Delta t_{\text{obs}}}{1 + z} = a_{LV} K + b_{sf}
\]

where

\[
K = \frac{1}{1 + z} \int_0^z \frac{dz}{H(z)}
\]

and

\[
a_{LV} = \frac{\Delta E}{\mathcal{M}_{QG}} \frac{1}{H_0}.
\]
Based on BATSE, HETE and SWIFT data it was shown that \( M_{QG} \geq 1.4 \times 10^{16} \text{GeV} \)

In 2005, MAGIC recorded VHE $\gamma$-ray flares of Mkn 501 ($z=0.034$). Based on these observations it was shown that \(^4\)

$$M_{\text{QG}} \geq 0.21 \times 10^{18} \text{GeV}$$

In 2006, HESS observed a giant outburst of the blazar PKS 2155-304 ($z=0.116$). Based on these observations it was shown that \(^5\)

$$M_{\text{QG}} \geq 0.72 \times 10^{18} \text{GeV}$$
