Asymptotic silence in quantum gravity

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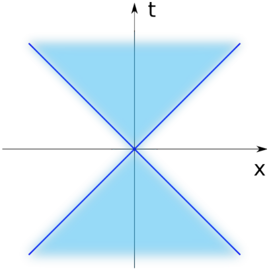
$c \to 0$



Minkowski

[1] J-M. Levy-Leblond, *Annales de l'I.H.P.*, section A, tome 3, No 1 (1965).

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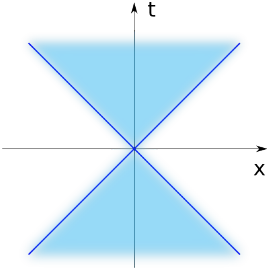
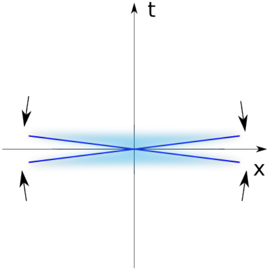


Minkowski



Newton ($c \to \infty$)

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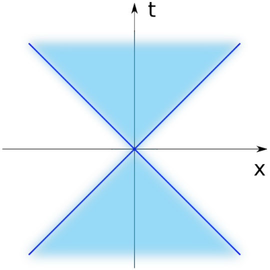
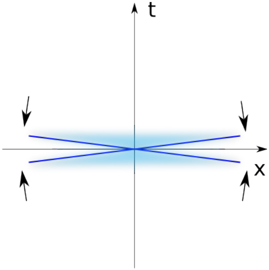


Minkowski



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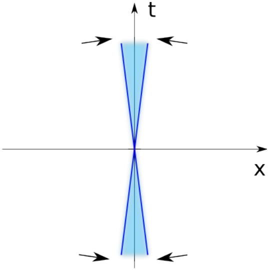
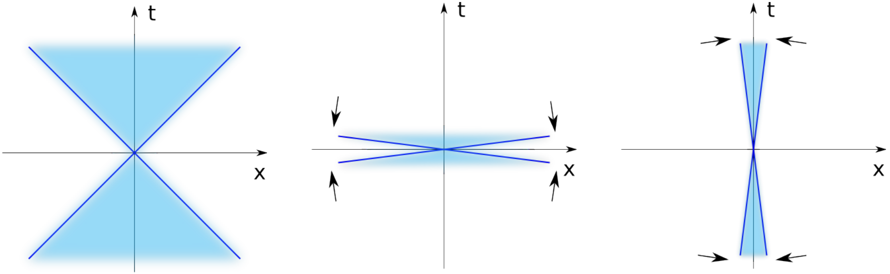


Carroll ($c \to 0$)

[1] J-M. Levy-Leblond, *Annales de l'I.H.P.*, section A, tome 3, No 1 (1965).

$c \to 0$



Minkowski   Newton ($c \to \infty$)   Carroll ($c \to 0$)

The asymptotic silence is obtained while taking the speed of light $c \to 0$, which is known as the Carrollian limit[1].

,,A slow sort of country ..., ... now, here, you see, it takes all the running you can do to stay in the same place..." Lewis Carroll

[1] J-M. Levy-Leblond, *Annales de l'I.H.P.*, section A, tome 3, No 1 (1965).

BKL conjecture

The state of asymptotic silence appears in various contexts. Perhaps the best known is the so-called Belinsky-Khalatnikov-Lifshitz (BKL) conjecture[2]:

"Near to a singularity spatially separated points decouple, and the role of most forms of matter is negligible."

---

[2] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, *Adv. Phys.* **19** (1970) 525; **31** (1982) 639

BKL conjecture

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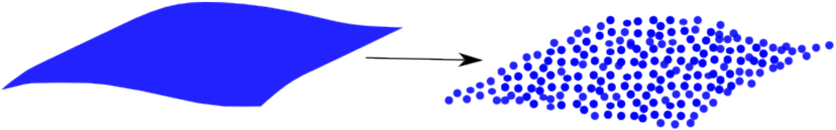


In the BKL scenario, each of the "points" is described by the anisotropic cosmological solution.

[2] V. A. Belinskii, I. M. Khalatnikov and E. M. Lifshitz, *Adv. Phys.* **19** (1970) 525; **31** (1982) 639

The less intuitive case is the strong coupling limit of the gravitational interactions, when $G \to \infty$. Relation between this limit and the asymptotic silence can be understood by analyzing the Hamiltonian formalism of general relativity, where

$$H_G[N, N^a] = S[N] + D[N^a] = \int d^3x (NS + N^a D_a).$$

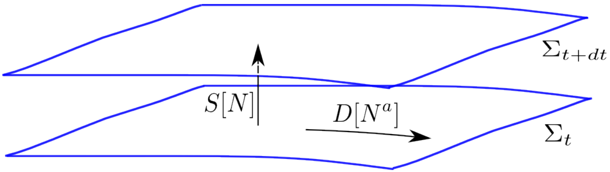
Namely, the scalar constraint can be schematically written as

$$S = G \cdot \ kinetic + \frac{1}{G} \cdot \ potential.$$

Here, only the potential term contains spatial derivatives, which relate the neighboring points. Therefore, while talking $G \to \infty$ only the kinetic term survives, and the theory becomes ultralocal.

Hypersurface deformation algebra

The constraints generate gauge transformations:



The algebra of constraints:

$$\begin{aligned}
\{D[N_1^a], D[N_2^a]\} &= D[N_1^b \partial_b N_2^a - N_2^b \partial_b N_1^a], \\
\{S[N], D[N^a]\} &= -S[N^a \partial_a N], \\
\{S[N_1], S[N_2]\} &= sD\left[g^{ab}(N_1 \partial_b N_2 - N_2 \partial_b N_1)\right],
\end{aligned}$$

where $s = 1$ corresponds to the Lorentzian signature and $s = -1$ to the Euclidean one. Due to the factor g^{ab} the algebra of constraint is not a Lie algebra.

Ultralocal gravity and Hořava-Lifshitz gravity

In the ultralocal limit[3] ($G \to \infty$) the algebra of constraints simplifies to the Lie algebra:

$$\begin{aligned} \{D[N_1^a], D[N_2^a]\} &= D[N_1^b \partial_b N_2^a - N_2^b \partial_b N_1^a], \\ \{S[N], D[N^a]\} &= -S[N^a \partial_a N], \\ \{S[N_1], S[N_2]\} &= 0. \end{aligned}$$

Surprisingly, the number of the local symmetry generators is the same as in GR. The same holds for Hořava-Lifshitz[4] gravity in the $z \to 0$ limit of the dynamical exponent, where the anisotropic scaling

$$\mathbf{x} \to b\mathbf{x} \quad \text{and} \quad t \to b^z t.$$

Cosmological evolution can be interpreted as a flow from $z = 0$ ($d_s \to \infty$) in the early universe to $z = 1$ ($d_s = 4$) observed now.

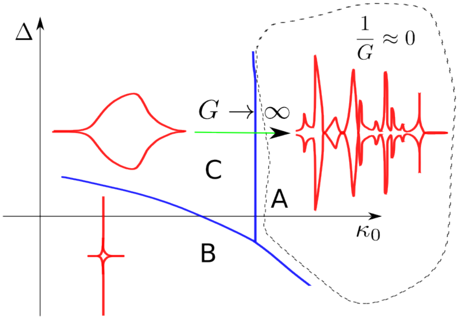
[3] C. J. Isham, *Proc. Roy. Soc. Lond. A* **351** (1976) 209.

[4] P. Hořava, *Phys. Rev. D* **79** (2009) 084008.

Causal Dynamical Triangulation

Possible confirmation of the asymptotic silence is coming also from Causal Dynamical Triangulation (CDT) approach to quantum gravity (Ambjorn, Jurkiewicz, Loll).

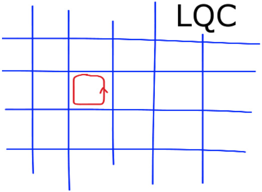
Phase diagram of CDT:



As observed from the numerical computations, universe breaks up into several independent components, when the effective gravitational coupling constant G increases.

Loop quantum gravity

- Loop quantum cosmology (LQG) is based on Hamiltonian formulation of GR.
- Loop Quantum Cosmology (LQC) is a regular lattice model of LQG.



- Physical area of a loop $\mathrm{Ar}_{\square} = \bar{p}\bar{\mu}^2$, where $\bar{p} = a^2$ and a is a scale factor. In general $\bar{\mu} \propto \bar{p}^{\beta}$, where $-1/2 \leq \beta \leq 0$. For the so-called $\bar{\mu}-$scheme: $\bar{\mu} = \sqrt{\frac{\Delta}{\bar{p}}}$, where $\Delta = 2\sqrt{3}\pi\gamma l_{\mathrm{Pl}}^2$ is the area gap derived from LQG.

- At the effective level, quantum gravity effects can be studied by introducing appropriate corrections to the classical constraints:

$$\mathcal{C}_{tot} \to \mathcal{C}^{Q}_{tot},$$

  where $\mathcal{C}_{tot} = \mathcal{C}_G + \mathcal{C}_M$.
- In LQC one usually consider two kinds of such quantum corrections:
  - Inverse volume corrections.
  - Holonomy corrections.
- Problems appear for inhomogeneous models:
- The procedure of introducing quantum corrections suffers from ambiguities.
- In general, the algebra of modified constraints is not closed:

$$\{\mathcal{C}^{Q}_{I}, \mathcal{C}^{Q}_{J}\} = g^{K}{}_{IJ}(A^{j}_{b}, E^{a}_{i})\mathcal{C}^{Q}_{K} + \mathcal{A}_{IJ}.$$

- Can we introduce quantum corrections in the anomaly-free manner (i.e. such that $\mathcal{A}_{IJ} = 0$)?
- It turns out that it is possible at least for perturbative inhomogeneities (application to cosmology) for:
  - Inverse volume corrections (gauge invariant: Bojowald, Hossain, Kagan, Shankaranarayanan - 2008)
  - Holonomy corrections (gauge invariant: Cailleteau, Mielczarek, Barrau, Grain - 2012, fixed gauge: Wilson-Ewing - 2012)
- We found that, for perturbative inhomogeneities with holonomy corrections:
  - There is a unique way of modifying constraints such that the algebra is closed.
  - Additionally, the conditions of anomaly-freedom are fulfilled if and only if $\mathrm{Ar}_{\Box}$ =const, which corresponds to "new quantization scheme".

See T. Cailleteau, J. Mielczarek, A. Barrau, J. Grain, Class. Quantum Grav. **29** (2012) 095010 and my PhD dissertation.

Algebra of constraints:

$$\begin{aligned}
\{D_{tot}[N_1^a], D_{tot}[N_2^a]\} &= 0, \\
\left\{S^Q_{tot}[N], D_{tot}[N^a]\right\} &= -S^Q_{tot}[\delta N^a \partial_a \delta N], \\
\left\{S^Q_{tot}[N_1], S^Q_{tot}[N_2]\right\} &= \beta D_{tot}\left[\frac{\bar{N}}{\bar{p}}\partial^a(\delta N_2 - \delta N_1)\right].
\end{aligned}$$

The algebra is closed but deformed with respect to the classical case due to presence of the factor

$$\beta = \cos(2\bar{\mu}\gamma\bar{k}) = 1 - 2\frac{\rho}{\rho_c} \in [-1, 1] \quad \text{where} \quad \rho_c = \frac{3}{8\pi G \Delta \gamma^2} \sim \rho_{\text{Pl}}.$$

What is the interpretation? Classically, we have

$$\{S_{tot}[N_1], S_{tot}[N_2]\} = sD\left[\frac{\bar{N}}{\bar{p}}\partial^a(\delta N_2 - \delta N_1)\right],$$

where $s = 1$ corresponds to the Lorentzian signature and $s = -1$ to the Euclidean one.

- The effective algebra of constraints shows that space is Euclidean for $\rho > \rho_c/2$, while Lorentzian geometry emerges for $\rho < \rho_c/2$. Signature change at $\rho = \rho_c/2$. Spacetime becomes 4D Euclidean space for $\rho > \rho_c/2$.
- It is interesting to notice that this model naturally have properties of the Hartle-Hawking no-boundary proposal (Hartle, Hawking - 1983).
- At $\rho = \rho_c/2$ the Ultralocal Gravity is recovered $\{S_{tot}[N_1], S_{tot}[N_2]\} = 0$ since $\beta = 0 \rightarrow$ the stage of asymptotic silence.
- The $\beta-$deformation appears also from the inverse-volume corrections. However, in that case $\beta > 0$. The asymptotic silence can be realized but not the signature change.
- However, the signature change was observed also for spherically symmetric models with holonomy corrections[5].

[5]M. Bojowald and G. M. Paily, *Phys. Rev. D* **86** (2012) 104018.

Equations of motion:

Scalar pertubations. One can derive modified Mukhanov equation:

$$\frac{d^2}{d\eta^2}v - \beta\nabla^2 v - \frac{z^{''}}{z}v = 0,$$

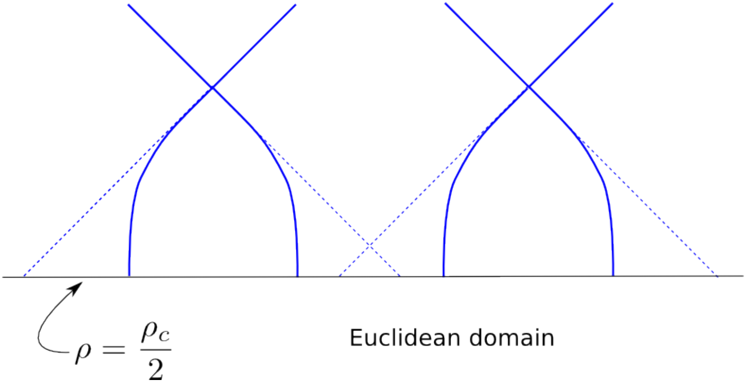
where $z := \sqrt{\bar{p}}\frac{\dot{\varphi}}{H}$. Spatial curvature $\mathcal{R} = v/z$.

Vector perturbations. For the considered model with a scalar field vector modes are pure gauge.

Tensor perturbations. Equation of motion for the gravitational waves is the following:

$$\frac{d^2}{d\eta^2}h_{ab} + 2\left(aH - \frac{1}{2\beta}\frac{d\beta}{d\eta}\right)\frac{d}{d\eta}h_{ab} - \beta\nabla^2 h_{ab} = 0.$$

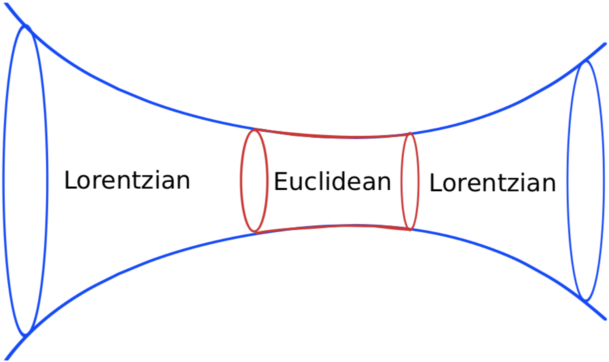
Light cones



Effective speed of light:

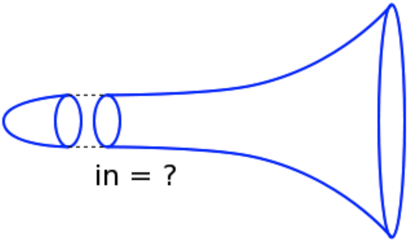
$$c_{eff} = \sqrt{\beta} = \sqrt{1 - 2\frac{\rho}{\rho_c}}.$$

The asymptotic silence ($c_{eff} \to 0$) is realized while $\rho \to \frac{\rho_c}{2}$.



- Is there quantum tunneling through the Euclidean phase from the contraction to expansion phase?
- Suppression of the spatial derivatives while $\{S, S\} \to 0$. Possible support for the BKL conjecture.
- What kind of initial/boundary conditions?

Hartle-Hawking no-boundary proposal



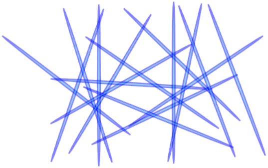
Probability amplitude:

$$\langle \text{out}|\text{in}\rangle = \int DgD\varphi e^{\frac{i}{\hbar}S}$$

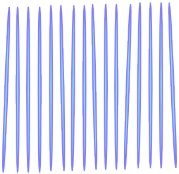
The no-boundary proposal is based on the assumption that the Wick rotation ($t \to -it$) gains the physical meaning at the Planck epoch.

Spontaneous symmetry breaking?

Let us consider a model of metamaterial, composed of magnetized nanowires (or ferromagnet, liquid crystal, etc.).



$T > T_C$



$T < T_C$

The $SO(3)$ symmetry is broken to $SO(2)$ at temperatures below T_C. An order parameter is the local magnetization and the corresponding Goldstone bosons manifest as spin waves.

In the direction of magnetization, the dielectric permittivity becomes negative leading to emergence of a new effective time variable [6]

[6] I. I. Smolyaninov and E. E. Narimanov, *Phys. Rev. Lett.* **105** (2010) 067402.

So, at the level of the equations of motion for the electric field propagating in the considered material, the original $SO(3)$ symmetry of Laplace operator is replaced by $SO(1,2)$.

In case of gravity, we observe that the symmetry of equations for the fields change from $SO(4)$ in the Euclidean region to $SO(1,3)$ in the Lorentzian regime. One can speculate that this transition is a result of the symmetry breaking at the level of the fundamental structure of spacetime.

In particular, one can suppose that the original $SO(4)$ spacetime symmetry is broken into $SO(3)$, where the residue $SO(3)$ is the rotational symmetry of triads. Time can be therefore seen as order parameter of the symmetry broken phase. Interestingly, in such a picture, Goldstone bosons associated with the broken symmetry must appear. Such particles could naturally serve as inflatons, which are required to explain the inflationary stage after the Planck epoch.

Summary and outlook

- Different approaches to quantum gravity meet at the asymptotic silence.
- Our model unifies various ideas, such as signature change and asymptotic silence.
- The presented results contribute to the growing evidence, showing that the asymptotic silence may indeed have something to do with the state of spacetime under very extreme conditions. However, the final vote belongs to experiment.
- Silent initial conditions at $\rho = \rho_c/2$. Predictions of the spectra of primordial perturbations. Comparison with the CMB data.
- Deformation of the Poincaré algebra? Possible modifications of the photon dispersion relation.
- Paradigm shift in LQC.