Towards solving generic cosmological singularity problem

Włodzimierz Piechocki

Department of Fundamental Research
National Centre for Nuclear Research
Warsaw, Poland

Quantum Cosmology Group:
Ewa Czuchry, Orest Hrycyna, Przemek Małkiewicz, Kuba Mielczarek, and WP

(National Centre for Nuclear Research)

OUTLINE

Introduction

Evidence for an existence of the cosmological singularity

- observational cosmology:
  the Universe has been expanding for almost 14 billion years
  (emerged from a state with extremely high energy densities
  of matter fields)
- theoretical cosmology:
  almost all known general relativity models of the Universe:
  (Lemaître, Kasner, AdS, Friedmann, Bianchi, ..., BKL)
  predict existence of cosmological singularities
  (blowing up gravitational and matter fields invariants)

Existence of the cosmological singularities in solutions to GR means that this classical theory is incomplete

Expectation: quantization may heal the singularity

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Introduction (cont)

Some intriguing questions concerning quantum phase:

- What is the energy scale?
- What is the nature of dark energy?
- What is the origin of cosmological inflation?
- What is the mechanism of the cosmological transitions: quantum phase $\rightleftarrows$ classical phase?
- What is the origin of tiny fluctuations visible in CMB?
- How long had the quantum phase lasted?
- What was before the Big Bounce?

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Two alternative methods of canonical quantization of cosmological models of GR, which are based on loop geometry

- Dirac's LQC[1] := 'first quantize then impose constraints'
- RPS LQC[2] := 'first solve constraints then quantize'

[1] Ashtekar, Bojowald, Lewandowski, Rovelli, Thiemann, ...
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Hamiltonian

(Einstein-Hilbert $\rightarrow$ Palatini $\rightarrow$ Holst)

$$H_g := \int_\Sigma d^3x(N^iC_i + N^aC_a + NC) \approx 0, \tag{1}$$

where

Σ, space-like part of spacetime $\mathbb{R}\times\Sigma$; (N^i, N^a, N), Lagrange multipliers; (C_i, C_a, C) are Gauss, diffeomorphism and scalar constraints; $(a, b = 1, 2, 3)$, spatial indices; $(i, j, k = 1, 2, 3)$ internal $SU(2)$ indices. Constrains must satisfy specific algebra.

For flat FRW universe with massless scalar field

$$H_g = -\gamma^{-2}\int_\mathcal{V} d^3x\; Ne^{-1}\varepsilon_{ijk}E^{aj}E^{bk}F^i_{ab}\,, \tag{2}$$

where

γ, Barbero-Immirzi parameter; $\mathcal{V}\subset\Sigma$, elementary cell; N, lapse function; ε_{ijk}, alternating tensor; E^a_i, density weighted triad; $F^k_{ab} = \partial_a A^k_b - \partial_b A^k_a + \epsilon^k_{ij}A^i_a A^j_b$, curvature of $SU(2)$ connection A^i_a; $e := \sqrt{|\det E|}$;

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Hamiltonian (cont)

Modification by loop geometry means approximation of F^k_{ab} :
Making use of the mean-value and Stokes' theorems we get

$$\tau_k\, F^k_{ab}(\vec{x}) \approx \frac{1}{s^\sigma_{ab}} \int_\sigma \tau_k\, F^k_{cd}\, dx^c \wedge dx^d = \frac{1}{s^\sigma_{ab}} \Big(\mathcal{P} \exp \big(\oint_{\partial\sigma} \tau_k\, A^k_c\, dx^c \big) - 1 \Big), \tag{3}$$

where $\partial\sigma$ is the boundary of a small surface σ with center at $\vec{x}$;
$s^\sigma_{ab} := \int_\sigma dx^a \wedge dx^b$; $\mathcal{P}$ denotes the path ordering symbol.
Choosing $\partial\sigma$ in the form of square $\Box_{ij}$ with sides length $\lambda V_0^{1/3}$ gives

$$F^k_{ab}(\lambda) \approx -2\ Tr \left(\frac{h^{(\lambda)}_{\Box_{ij}} - 1}{\lambda^2 V_o^{2/3}} \right) \tau^k\ {}^o\omega^i_a\ {}^o\omega^j_a, \qquad F^k_{ab} = \lim_{\lambda \to 0} F^k_{ab}(\lambda), \tag{4}$$

where $h^{(\lambda)}_{\Box_{ij}}$ is a holonomy of connection around the loop $\Box_{ij}$,
and where λ is a size of the loop.

Hamiltonian (cont)

Holonomy $h_s(A)$ of connection A along curve $s : [0,1] \rightarrow s(t) \in \Sigma$ is the solution to the equation

$$\frac{d}{dt} h_{s_t}(A) = A(s_t) h_{s_t}(A), \quad h_{s_0} = \mathbb{I}, \tag{5}$$

where $A(s_t) := A^j_a(s_t)\, \tau_j\, \dot{s}^a(t)$, $h_s(A) := h_{s_1}(A) \in SU(2)$, $s_t \equiv s(t)$.

Holonomy along straight edge ${}^o e^a_k \partial_a$ of length $\lambda V_0^{1/3}$ (in fundamental, j = 1/2, representation of SU(2) group) reads

$$h_k^{(\lambda)}(c) = \exp(\tau_k \lambda c) = \cos(\lambda c/2)\, \mathbb{I} + 2 \sin(\lambda c/2)\, \tau_k, \tag{6}$$

where $A^k_a = {}^o\omega^k_a\, c\, V_0^{-1/3}$ and $\tau_k = -i\sigma_k/2$ (σ_k are the Pauli spin matrices)

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Making use of Thiemann's identity leads finally to

$$H_g = \lim_{\lambda \to 0} H_g^{(\lambda)}, \tag{7}$$

$$H_g^{(\lambda)} = -\frac{sgn(p)}{2\pi G \gamma^3 \lambda^3} \sum_{ijk} N \varepsilon^{ijk}\, Tr\Big(h_i^{(\lambda)} h_j^{(\lambda)} (h_i^{(\lambda)})^{-1} (h_j^{(\lambda)})^{-1} h_k^{(\lambda)} \{(h_k^{(\lambda)})^{-1}, V\}\Big) \tag{8}$$

and where $V = |p|^{\frac{3}{2}} = a^3 V_0$ is the volume of the elementary cell $\mathcal{V}$.
Variables c and p determine connections A^k_a and triads E^a_k:
$A^k_a = {}^o\omega^k_a\, c\, V_0^{-1/3}$ and $E^a_k = {}^o e^a_k \sqrt{q_o}\, p\, V_0^{-2/3}$,
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Remark! Up to this point there is no difference between Dirac's and RPS LQC methods.

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Hamiltonian (cont)

Total Hamiltonian for FRW universe with a massless scalar field ϕ

$$H = H_g + H_\phi, \quad H_\phi := p_\phi^2 \, |p|^{-\frac{3}{2}}/2 \tag{9}$$

where ϕ and p_ϕ are elementary variables satisfying $\{\phi, p_\phi\} = 1$.
The relation $H \approx 0$ enables defining physical phase space.
Making use of (6) we calculate (8) and get the modified total Hamiltonian corresponding to (9)

$$H^{(\lambda)}/N = -\frac{3}{8\pi G\gamma^2} \, \frac{\sin^2(\lambda\beta)}{\lambda^2} \, v + \frac{p_\phi^2}{2\,v}, \tag{10}$$

where the canonical variables read

$$\beta := \frac{c}{|p|^{1/2}}, \quad v := |p|^{3/2}, \tag{11}$$

where $\beta \sim \dot{a}/a$ and $v \sim a^3$, for $\lambda = 0$.
Eq (10) presents a modified classical Hamiltonian with a free parameter λ to be determined from observational data.
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Elementary observables

A function $\mathcal{O}$ defined on phase space is called the Dirac observable if it weakly Poisson commutes with the constraint, i.e.

$$\{\mathcal{O}, H^{(\lambda)}\} \approx 0. \tag{12}$$

The sign '≈ 0' means that we first make the Poisson bracket, and then impose the constraint $H^{(\lambda)} = 0$

In practice, we first solve (12) in a strong '$=$' form:

$$\frac{\sin(\lambda\beta)}{\lambda}\frac{\partial\mathcal{O}}{\partial\beta} - v\cos(\lambda\beta)\frac{\partial\mathcal{O}}{\partial v} - \frac{\kappa\gamma\,\mathrm{sgn}(p_\phi)}{4\pi G}\frac{\partial\mathcal{O}}{\partial\phi} = 0, \tag{13}$$

and then impose the constraint on the solutions to (13).

Elementary observables := simplest solutions to (13) are

$$\mathcal{O}_1 := p_\phi, \quad \mathcal{O}_2 := \phi - \frac{\mathrm{sgn}(p_\phi)}{3\kappa}\,\mathrm{arth}\big(\cos(\lambda\beta)\big), \quad \mathcal{O}_3 := \mathrm{sgn}(p_\phi)\, v\,\frac{\sin(\lambda\beta)}{\lambda}. \tag{14}$$

Elementary observables satisfy the Lie algebra

$$\{\mathcal{O}_2, \mathcal{O}_1\} = 1, \quad \{\mathcal{O}_1, \mathcal{O}_3\} = 0, \quad \{\mathcal{O}_2, \mathcal{O}_3\} = \gamma\kappa. \tag{15}$$

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Elementary observables (cont)

Due to the constraint $H^{(\lambda)} = 0$, we have

$$\mathcal{O}_3 = \gamma\kappa\,\mathcal{O}_1. \tag{16}$$

One can show that in the physical phase space, $\mathcal{F}^{(\lambda)}_{phys}$, we have only two observables which satisfy the algebra

$$\{\mathcal{O}_2, \mathcal{O}_1\} = 1, \tag{17}$$

where

$$\{\cdot,\cdot\} := \frac{\partial\cdot}{\partial\mathcal{O}_2}\frac{\partial\cdot}{\partial\mathcal{O}_1} - \frac{\partial\cdot}{\partial\mathcal{O}_1}\frac{\partial\cdot}{\partial\mathcal{O}_2}. \tag{18}$$

$\mathcal{F}^{(\lambda)}_{kin}$ is four dimensional. In relative dynamics one variable is used to parametrize three others. Since the constraint relates two variables, we have only two independent variables.

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Compound observables

Compound observables are functions of elementary observables. They are supposed to be measurable observables.

In what follows we consider two compound observables

- the volume in space

$$v(\phi, \lambda) = \kappa\gamma\lambda\,|\mathcal{O}_1|\,\cosh 3\kappa(\phi - \mathcal{O}_2) \tag{19}$$

- the energy density of matter field

$$\rho(\phi, \lambda) = \frac{1}{2}\,\frac{1}{(\kappa\gamma\lambda)^2 \cosh^2 3\kappa(\phi - \mathcal{O}_2)} \tag{20}$$

For fixed ϕ both ρ and v are the Dirac observables.

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Volume operator

Classical volume operator, v, is defined in a standard way as follows

$$v := \int_{\mathcal{V}} dx_1 dx_2 dx_3 \sqrt{\det q_{ab}} = a^3 \int_{\mathcal{V}} dx_1 dx_2 dx_3 \sqrt{\det q^0_{ab}} =: a^3\, V_0, \tag{21}$$

where $\mathcal{V} \subset \Sigma$ is an elementary cell in the space with topology $\mathbb{R}^3$; $(x_a) = (x^a) = (x^1, x^2, x^3)$ are Cartesian coordinates; $q_{ab} := a^2\, q^0_{ab}$ is a physical 3-metric; a is a scale factor; $q^0_{ab} dx^a dx^b := dx_1^2 + dx_2^2 + dx_3^2$ defines a fiducial 3-metric; V_0 is a fiducial volume (it does not occur in final results).

Volume operator (cont)

In terms of elementary observables the classical volume reads

$$v = |w|, \quad w := \kappa\gamma\lambda\, \mathcal{O}_1\, \cosh 3\kappa(\phi - \mathcal{O}_2). \tag{22}$$

Thus, quantization of v reduces to the quantization problem of w:

$$\hat{w}\, f(x) := \kappa\gamma\lambda\, \frac{1}{2}\left(\widehat{\mathcal{O}}_1\, \cosh 3\kappa(\phi - \widehat{\mathcal{O}}_2) + \cosh 3\kappa(\phi - \widehat{\mathcal{O}}_2)\, \widehat{\mathcal{O}}_1\right) f(x), \tag{23}$$

where $f \in L^2(\mathbb{R})$.

For $\mathcal{O}_1$ and $\mathcal{O}_2$ we use the Schrödinger representation:

$$\mathcal{O}_1 \longrightarrow \widehat{\mathcal{O}}_1 f(x) := -i\,\hbar\,\partial_x f(x), \quad \mathcal{O}_2 \longrightarrow \widehat{\mathcal{O}}_2 f(x) := \widehat{x} f(x) := x f(x). \tag{24}$$

Thus, an explicit form of $\hat{w}$ is

$$\hat{w} = i\,\frac{\kappa\gamma\lambda\hbar}{2}\Big(2\cosh 3\kappa(\phi - x)\,\frac{d}{dx} - 3\kappa\sinh 3\kappa(\phi - x)\Big). \tag{25}$$

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Volume operator (cont)

Solution to the eigenvalue problem:

$$\hat{w}\, f_a(x) = a\, f_a(x), \quad a \in \mathbb{R}, \tag{26}$$

$$f_a(x) := \frac{\sqrt{\frac{3\kappa}{\pi}}\, \exp\left(i \frac{2a}{3\kappa^2 \gamma \lambda \hbar} \arctan e^{3\kappa(\phi - x)}\right)}{\cosh^{\frac{1}{2}} 3\kappa(\phi - x)}, \tag{27}$$

$$a = b + 6\kappa^2 \gamma \lambda \hbar\, m = b + 8\pi G \gamma \lambda \hbar\, m, \tag{28}$$

where $b \in \mathbb{R}$ and $m \in \mathbb{Z}$.
Completion of the span of

$$\mathcal{F}_b := \{\, f_a \mid a = b + 8\pi G \gamma \lambda \hbar\, m \} \subset L^2(\mathbb{R}), \tag{29}$$

in the norm of $L^2(\mathbb{R})$ leads to $L^2(\mathbb{R})$, $\forall b \in \mathbb{R}$.
The operator $\hat{w}$ is essentially self-adjoint on each span of $\mathcal{F}_b$.

Volume operator (cont)

Due to the the relation (22) and the spectral theorem on self-adjoint operators we get the solution of the eigenvalue of the volume operator

$$v = |w| \quad \longrightarrow \quad \hat{v} f_a := |a| f_a. \tag{30}$$

The spectrum is bounded from below and discrete.

There exists the minimum gap $\triangle := 8\pi G\gamma\hbar\lambda$ in the spectrum, which defines a quantum of the volume.

In the limit $\lambda \rightarrow 0$, corresponding to the classical FRW model, there is no quantum of the volume.

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Energy density operator

The energy density operator reads

$$\widehat{\rho} := \frac{1}{2(\kappa\gamma\lambda)^2 \cosh^2 3\kappa(\phi + i\,\hbar\,\partial_x)}. \qquad (31)$$

The spectral theorem leads to

$$\widehat{\rho}\, f_p = \rho(p)\, f_p, \quad \rho(p) := \frac{1}{2(\kappa\gamma\lambda)^2 \cosh^2 3\kappa(\phi - p)} \qquad (32)$$

- energy density is well defined for any $\phi \in \mathbb{R}$
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Energy density operator (cont)

The density ρ has maximum at the minimum of v:

$$\rho_{\text{max}} = \frac{1}{2\kappa^2\gamma^2}\,\frac{1}{\lambda^2}. \tag{33}$$

For the Planck scale:
substituting $\lambda = l_{Pl}$ into (33) gives $\rho_{max}/\rho_{Pl} \simeq 2,07$;
for $\rho_{\text{max}} = \rho_{Pl}$ we get $\lambda \simeq 1,44\ l_{Pl}$.

Equation (33) fits the Planck scale!

- The spectrum is continuous and bounded, $(0, \frac{1}{2(\kappa\gamma\lambda)^2})$, for fixed λ.
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Evolution of quantum system

To define an evolution of of the universe in a quantum phase we identify first the so-called true Hamiltonian, H. It is obtain by inserting the constraint into Hamilton's equations, and finding a new Hamiltonian that generates the dynamics. One gets

$$H = \frac{2}{\lambda\sqrt{G}} P \sin(\lambda Q), \tag{34}$$

where $P := v/(4\pi l_{\mathrm{Pl}}^2 \gamma)$ and $Q := \beta$, and where $\{Q, P\} = 1$.

In the Schrödinger representations for these variables

$$\hat{Q}\phi(Q) := Q\phi(Q), \qquad \hat{P}\phi(Q) := -i\frac{d}{dQ}\phi(Q). \tag{35}$$

Quantum Hamiltonian corresponding to (34) reads

$$\hat{H}_\lambda \psi = -\frac{i}{\lambda\sqrt{G}} \left(2\sin(\lambda Q)\frac{d}{dQ} + \lambda\cos(\lambda Q) \right) \psi, \tag{36}$$

where $\psi \in D \subset \mathcal{H} := L^2([0, \pi/\lambda], dQ)$, and where D is some dense subspace of $\mathcal{H}$.

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Evolution (cont)

The eigenvalue problem, $\hat{H}_\lambda \Psi_E = E\Psi_E$, has the solution

$$\Psi_E(x) = \sqrt{\frac{\lambda\sqrt{G}}{4\pi}\cosh\left(\frac{2}{\sqrt{G}}x\right)}e^{iEx}, \quad E \in \mathbb{R}, \tag{37}$$

where $x := \frac{\sqrt{G}}{2}\ln\left|\tan\left(\frac{\lambda Q}{2}\right)\right|$.

We specify the domain of $\hat{H}_\lambda$ as follows

$$D(\hat{H}_\lambda) := \mathrm{span}\{\varphi_k, \; k \in \mathbb{Z}\}, \tag{38}$$

where

$$\varphi_k(Q) := \int_{-\infty}^{\infty} c_k(E)\Psi_E(Q)\, dE, \quad c_k \in C_0^\infty(\mathbb{R}). \tag{39}$$

One can prove that $\hat{H}_\lambda$ is an essentially self-adjoint operator on $D(\hat{H}_\lambda)$.

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Evolution (cont)

The classical Hamiltonian H is positive-definite because $\lambda \in [0, \pi]$ and $P \in [0, \infty)$. The corresponding self-adjoint operator $\hat{H}$ has however eigenvalues $E \in \mathbb{R}$. We therefore introduce a physical Hamiltonian $\hat{\mathbb{H}}$, which has only nonnegative eigenvalues. It is defined as follows

$$\hat{\mathbb{H}}\Psi_E := |E|\Psi_E, \quad E \in \mathbb{R}. \tag{40}$$

Using Stone's theorem we define an unitary operator of the evolution

$$\hat{U}(s) = e^{-is\hat{\mathbb{H}}}, \tag{41}$$

where $s \in \mathbb{R}$ is a time parameter. The state at any moment of time reads

$$|\Psi(s)\rangle = \hat{U}(s)|\Psi(0)\rangle = e^{-is\hat{\mathbb{H}}}|\Psi(0)\rangle. \tag{42}$$

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Evolution (cont)

Let us consider a superposition of the Hamiltonian eigenstates $|\Psi(0)\rangle = \int_{-\infty}^{+\infty} dEc(E)|\Psi_E\rangle$ at $s = 0$. Then, evolution of this state is given by

$$|\Psi(s)\rangle = \int_{-\infty}^{+\infty} dEc(E)e^{-isE}|\Psi_E\rangle. \tag{43}$$

We consider the Gaussian packet with a simple profile defined to be $c(E) := \sqrt[4]{2\alpha/\pi}\exp\left\{-\alpha(E - E_0)^2\right\}$, that is centered at E_0 with the dispersion parametrized by α. The normalized packet corresponding to (43) reads

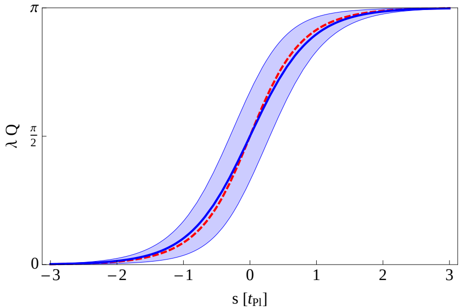
$$\Psi(x, s) = \sqrt{\frac{\lambda \cosh\left(\frac{2}{\sqrt{G}}x\right)}{\sqrt{8\pi\tilde{\alpha}}}} e^{-\frac{(x-s)^2}{4\alpha}} e^{iE_0(x-s)}, \tag{44}$$

where $\tilde{\alpha} := \alpha/G$.

Evolution (cont)

The dispersion of $\hat{Q}$ in the state $|\Psi(s)\rangle$ reads

$$\Delta\hat{Q} := \sqrt{\langle\Psi(s)|\hat{Q}^2|\Psi(s)\rangle - (\langle\Psi(s)|\hat{Q}|\Psi(s)\rangle)^2}. \qquad (45)$$

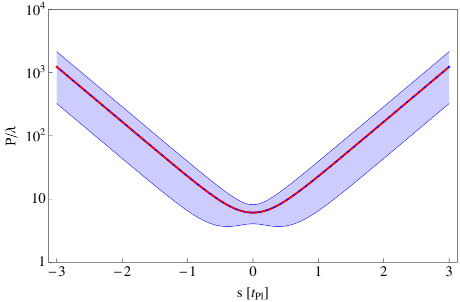


Blue line represents mean value $\langle\hat{Q}\rangle$. Red line is the classical solution. Shadowed region means $\langle\hat{Q}\rangle \pm \Delta\hat{Q}$. Maximum of $\Delta\hat{Q}$ occurs at the bounce, $s = 0$.

Evolution (cont)

The dispersion of $\hat{P}$ in the state $|\Psi(s)\rangle$ reads

$$\Delta\hat{P} := \sqrt{\langle\Psi(s)|\hat{P}^2|\Psi(s)\rangle - (\langle\Psi(s)|\hat{P}|\Psi(s)\rangle)^2} \qquad (46)$$



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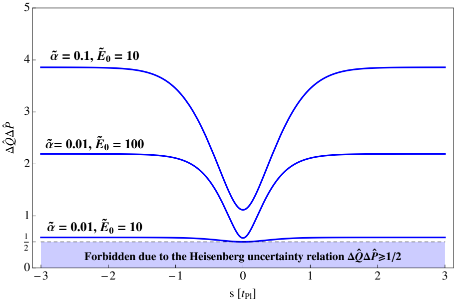
Evolution (cont)



Evolution of the product of dispersions $\Delta\hat{Q}\,\Delta\hat{P}$. Heisenberg's uncertainty relation $\Delta\hat{Q}\,\Delta\hat{P} \geq 1/2$ is preserved during the evolution. Minimum occurs at the bounce, $s = 0$.

Conclusions

- Cosmic singularitry problem of FRW model can be solved by using the loop geometry: big bang turns into big bounce
- Discreteness of the spectra of the volume operators may favor a foamy structure of space at short distances that may be detected in astro-cosmo observations
  - cosmic photons: no dispersion of photons[3] up to the energy 5×10^{17} GeV
  - detection of primordial gravity waves: imprints of tensor modes on CMB spectrum[4]
- Evolution of a quantum phase can be described in terms of the self-adjoint physical Hamiltonian
  - expectation values of quantum variables coincide with corresponding classical variables
  - Heisenberg's uncertainty relation is perfectly satisfied during the entire evolution of the universe.

[3] F. Aharonian *et al.*, Phys. Rev. Lett. **101**, 170402 (2008), 'Limits on an Energy Dependence of the Speed of Light from a Flare ...'

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Main papers:

[1] P. Dzierżak, P. Małkiewicz and W. P.,
'Turning big bang into big bounce: I. Classical dynamics',
Phys. Rev. D **80**, 104001 (2009).

[2] P. Małkiewicz and W. P.,
'Turning big bang into big bounce: II. Quantum dynamics'.
Class. Quant. Grav. **27** (2010) 225018.

[3] P. Małkiewicz and W. P.,
'Energy scale of the Big Bounce',
Phys. Rev. D **80**, 063506 (2009).

[4] J. Mielczarek and W. P.,
'Observables for FRW model with cosmological constant'.
Phys. Rev. D **82** (2010) 043529.

[5] J. Mielczarek and W. P.,
'Evolution in bouncing quantum cosmology',
Class. Quant. Grav. **29** (2012) 065022.

[6] J. Mielczarek and W. P.,
'Gaussian state for the bouncing quantum cosmology ',
Phys. Rev. D **86** (2012) 083508.

[7] J-P. Gazeau, J. Mielczarek and W. P.,
'Quantum states of the bouncing universe',
in preparation.

Great challenge:

Quantization of the Belinskii-Khalatnikov-Lifshitz scenario (1963-82).

- FRW metric is dynamically unstable in the evolution towards the singularity (breaking of isotropy)
- Bianchi type metric is dynamically unstable in the evolution towards the singularity (breaking of homogeneity)
- BKL scenario (asymptotic silence) - solution of GR near CS
- BKL: does not rely on any symmetry conditions; general $:=$ corresponds to non-zero measure subset of all initial conditions; stable $:=$ solution is stable against perturbation of initial conditions
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- BKL: does not rely on any symmetry conditions; general $:=$ corresponds to non-zero measure subset of all initial conditions; stable $:=$ solution is stable against perturbation of initial conditions
- BKL appears in the low energy limit of superstring models
- application of non-singular quantum BKL theory
  - realistic model of the very early Universe
  - may help in the construction of the theory unifying gravitation and quantum physics.

Prospects

- quantum Bianchi I cosmology[5]
- quantum Bianchi II cosmology[6]
- diagonal Bianchi IX cosmology[7]
- non-diagonal Bianchi IX cosmology[8]
- isotropisation of the Bianchi type models[9]
- primordial gravity waves from the BB of the quantum Bianchi I model[10]
- doubtful if LQC/LQG can be used to resolve the singularity problem of the BKL theory[11].

[5] P. Dzierżak and W. P., Phys. Rev. D **80** (2009) 124033; P. Małkiewicz, P. Dzierżak, and W. P., Class. Quant. Grav. **28** (2011) 085020; P. Małkiewicz, Class. Quant. Grav. **29** (2012) 075008.

[6] QCG, in progress.

[7] P. Małkiewicz, in progress.

[8] E. Czuchry and WP, arXiv:1202.5448 [gr-qc].

[9] O. Hrycyna and WP, in progress.

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Thank you!